INTERNATIONAL JOURNAL FOR NUMERICAL METHODS IN FLUIDS, VOL. 18, 545–553 (1994)

ON ESTIMATING PRESSURE BUILD-UP IN SEVERELY ACCELERATED, SHOCKED FLOW*

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SUMMARY

A new approach to the calculation of the high pressures characterizing the flow field in front of a piston undergoing severe acceleration over the short term is presented. In contrast with previous approaches where the computational domain is altered but which stop short of transforming velocities, here the problem is solved in an accelerating non-Euclidean co-ordinate system where the piston is stationary. The method is applied to a study of the problem of premature sabot separation. Through use of Harten's second-order-accurate TVD scheme, flow simulations are performed for both 1D and 3D axisymmetric geometries. The simple 1D model gives pressure profiles surprisingly close to those of the more physical 3D model.

KEY WORDS Sabot separation Non-Euclidean co-ordinates Accelerated flow

1. INTRODUCTION

The problem of numerically calculating the flow in front of a non-uniformly accelerating piston has been studied by Moretti.[1] However, should it be desired to apply the method to axisymmetric flows, shock fitting is not well-suited and cases can arise where flow particulars differ from those previously assumed. A more recent treatment of the compressing piston problem occurs in Reference 2, where other approaches which use shock-capturing methods are given. However, it is not at all clear how to extend these methods to higher-dimensional flows and non-blunt projectiles.

Furthermore, one drawback encountered is that there arises from operator splitting the requirement to solve a convection equation

$$U_t - CU_x = 0, \tag{1}$$

where C is the speed of the piston. Although equation (1) has the exact solution

$$U = f(x + Ct), \tag{2}$$

delicacy is required when the initial profile is represented graphically. As the shock produced by the accelerating piston grows in strength, with finite difference methods overshoots eventually occur, which can be avoided by resort to TVD or ENO schemes.

In order to avoid solving the convection equation (1) in the presence of a strong shock, a new approach is sought. By transforming both velocities and computational domain, the full potential of the transformation technique can be achieved. This leads to solving a fluid flow problem

* Selected paper from the 'BAIL VI Conference, Colorado, U.S.A. 1992'—Special Editor: Professor J. H. H. Miller.

CCC 0271–2091/94/060545–09

Received November 1992
Revised August 1993

formulated in a non-Euclidean space. Of all approaches considered,[1,2] this method promises the easier generalization to higher-dimensional problems. For the problem of premature sabot separation, 1D results equivalent to those of Reference 2 are obtained. Results from the 1D model turn out surprisingly close to those obtained when 3D axisymmetric flow is simulated.

2. GOVERNING EQUATIONS

The conservation law form for the Euler equations which govern the one-dimensional, ideal compressible flow of a fluid is

$$U_t + F(U)_x = 0, \tag{3}$$

where

$$U = \begin{bmatrix} \rho \\ \rho u \\ \rho E \end{bmatrix} \tag{4}$$

and

$$F = \begin{bmatrix} \rho u \\ P + \rho u^2 \\ (P + \rho E)u \end{bmatrix}. \tag{5}$$

The unknowns are pressure P, density ρ, velocity u, specific internal energy e and specific total energy $E = e + u^2/2$. The independent variables are time t and streamwise distance x. For compressible flow of an ideal gas the equation of state $P = (\gamma - 1)\rho e$ applies.

3. TRANSFORMATION TO NON-EUCLIDEAN GEOMETRY

In Reference 2 the motion of the accelerating piston is arrested by a co-ordinate transformation which changes equation (1) to the form

$$U_t + F(U)_x - u_p U_x = 0, \tag{6}$$

where $u_p = b(t)$ is the velocity of the accelerating piston. The computational domain has been altered, yet the physical fluid velocities are unchanged. Thus the motion of the piston progressing into the fluid is accounted for by a backward convection of the entire flow into the now stationary piston. To avoid the convection term appearing in equation (6), the further transformation

$$\tilde{u} = u - u_p \tag{7}$$

on the fluid velocities leads, after separation and recombination, to the fluid equations in a moving, non-Euclidean co-ordinate system. Actually, what is done to separate equations (3)–(5) into primitive variable form, invoke (7) and recombine in the usual fashion to produce the conservation law form

$$\tilde{U}_t + F(\tilde{U})_x + G = 0. \tag{8}$$

Here

$$G = \begin{bmatrix} 0 \\ \rho a_p \\ \rho u a_p \end{bmatrix}, \tag{9}$$

where a_p is the acceleration of the piston. The specific total energy in the non-Euclidean system is given by

$$\tilde{E} = e + 0{\cdot}5\tilde{u}^2, \tag{10}$$

whereas e, P and ρ are invariant.

Operator splitting

Yanenko's method[3] of operator splitting is used to numerically solve equations (8)–(10). A flow field update over time increment δt proceeds according to the relation

$$U_{n+1} = (T_H)(T_A)U_n. \tag{11}$$

The operator T_A takes into account the effects of the acceleration of the co-ordinate system represented by the term G. The operator T_H represents discarding the acceleration term G and updating the counterpart of equation (3) using Harten's second-order-accurate TVD scheme.[4] Since it is intended to compare results with the second method of Reference 2, where a first-order splitting is used, the above first-order splitting is employed.

Actually, for the case of an initial stagnation flow the flux derivative term in equation (8) vanishes and the resulting equations are exactly solved by holding ρ and e constant and updating the velocity over a short time interval δt according to equation (7) with u_p replaced by an appropriate velocity increment $\int a_p \delta t = \delta u$. Thus in the splitting the effects of acceleration can be exactly calculated.

4. BOUNDARY CONDITION TREATMENT

One approach to handling the wall boundary condition at the moving piston is that of Widhopf *et al.*,[5] who use a wave treatment of the near-wall physics to estimate the pressure at the piston. When the flow is compressive, a shock wave is fitted between the wall state and the state at the nearest grid point. Otherwise an expansion wave is fitted between these two fluid states. Alternatively, the wall pressure can be estimated by linear extrapolation employing pressures at the two nearest grid locations:

$$P_w = P_1 - 2P_2. \tag{12}$$

In Reference 2 these methods are found to be essentially equivalent. Thus the wave treatment will be used exclusively.

5. NUMERICAL RESULTS FOR A GIVEN ACCELERATION HISTORY

Jacketed projectiles undergoing in-bore acceleration to high velocities may experience premature separation of the sabot.[6] In this section we conduct a study of the high pressures characterizing the flow field in front of a projectile undergoing short-term, severe acceleration in the high supersonic range. Assuming zero blow-by of the propellant gas, perhaps the simplest tractable model for the high pressure build-up is that of one-dimensional flow in front of an accelerating piston. The piston is driven according to a measured velocity history experienced by a saboted projectile which is known to encounter premature in-bore separation.

However, in order to get some idea of the applicability of the 1D model, in the sequel axisymmetric flow models and non-blunt projectiles will also be considered. As it turns out, comparison of results appears to indicate that the small difference in accuracy is probably not worth the extra effort: the 1D model is sufficient for estimating the pressure build-up.

Figure 1 shows a typical velocity history for the firing of a saboted projectile during the time that 24 ft of in-bore motion occurs.[6] In-bore initial conditions before firing are atmospheric pressure and a temperature of 540 °R. Numerical results for this problem are now presented.

Figures 2–4 show the Mach number, pressure and velocity profiles in front of the piston at the time that approximately 24 ft in-bore have been traversed. Figure 5 shows the pressure build-up as a function of the distance traversed by the piston. To obtain the input piston velocity profile, a piecewise-quadratic fit to graphical velocity data is used. Since no trouble was taken to use a quadratic spline, time-wise discontinuities in acceleration lead to small but noticeable slope discontinuities in pressure. Figure 6 shows a comparison of results between the present method and the second method of Reference 2, where only the computational domain is transformed. There is clearly negligible difference in results for the two methods.

One minimal check of the validity of the present algorithm is to determine whether the flow in front of a piston moving at uniform velocity is reproduced. An affirmative result (not shown here) has been obtained computationally. Moreover, the results of Figures 2–6 are essentially unaffected by mesh refinement.

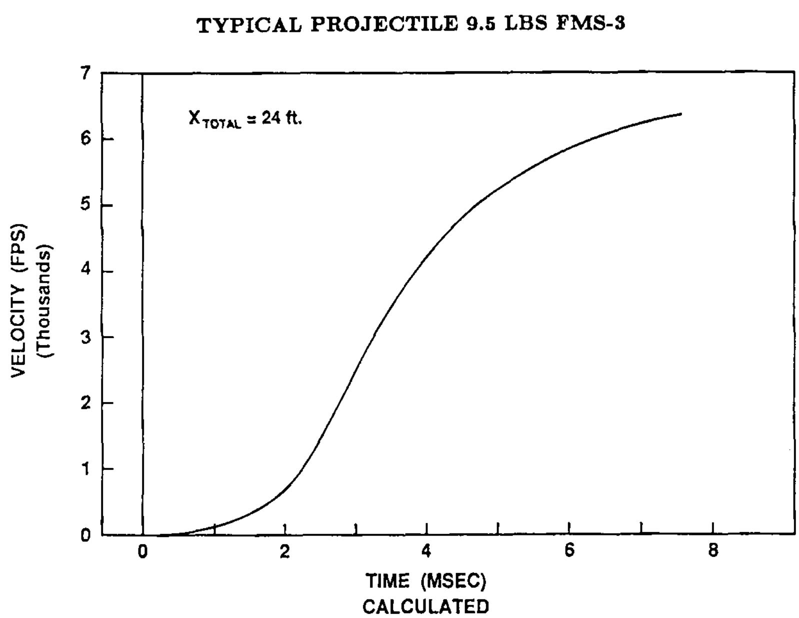

Figure 1. Projectile input velocity

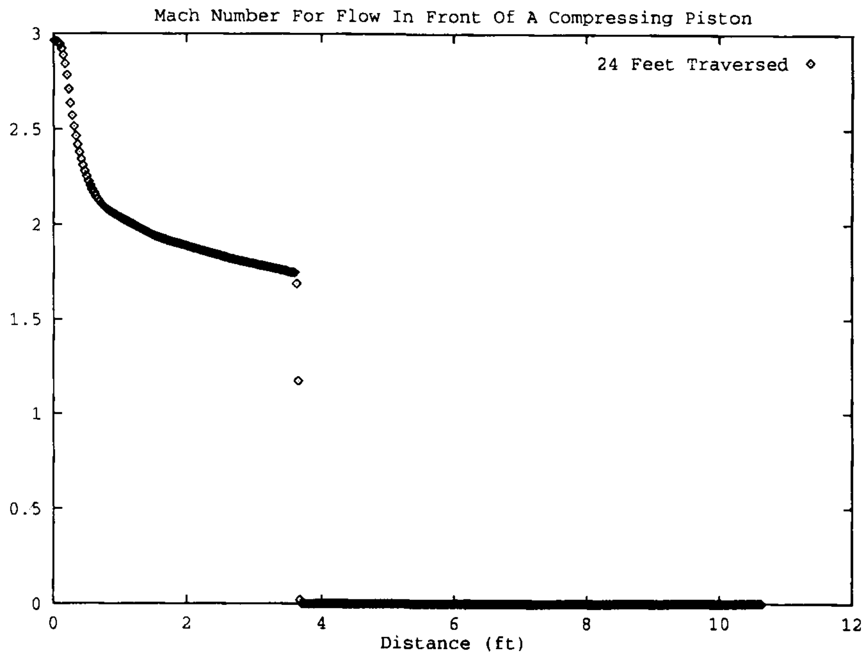

Figure 2. Exit Mach number profile

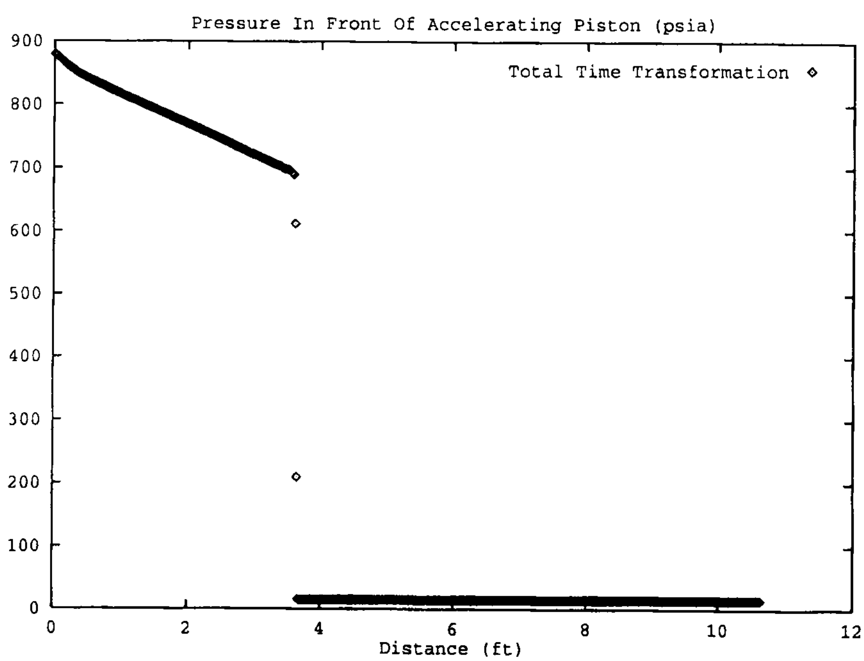

Figure 3. Exit pressure profile

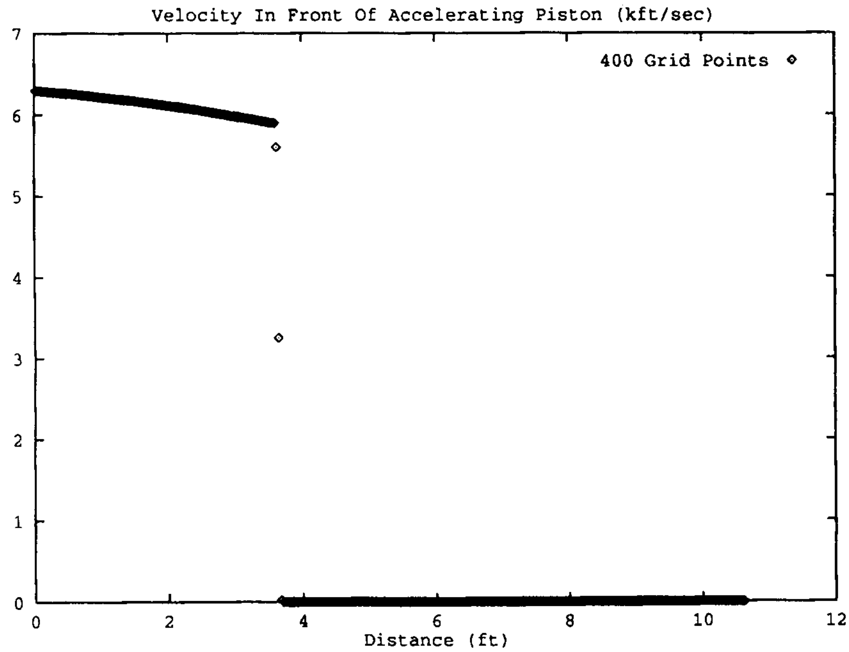

Figure 4. Exit velocity profile

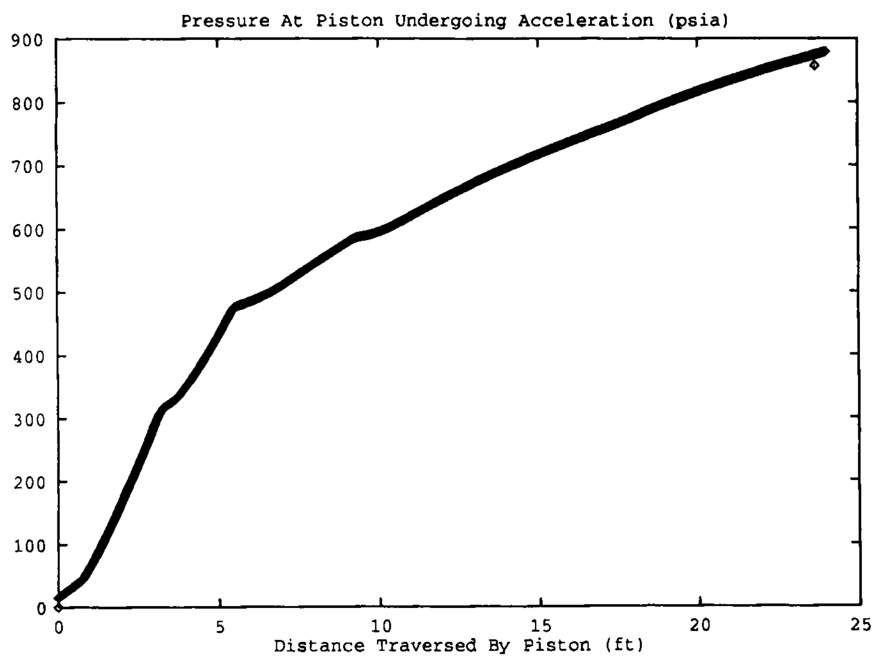

Figure 5. Piston pressure build-up versus distance travelled

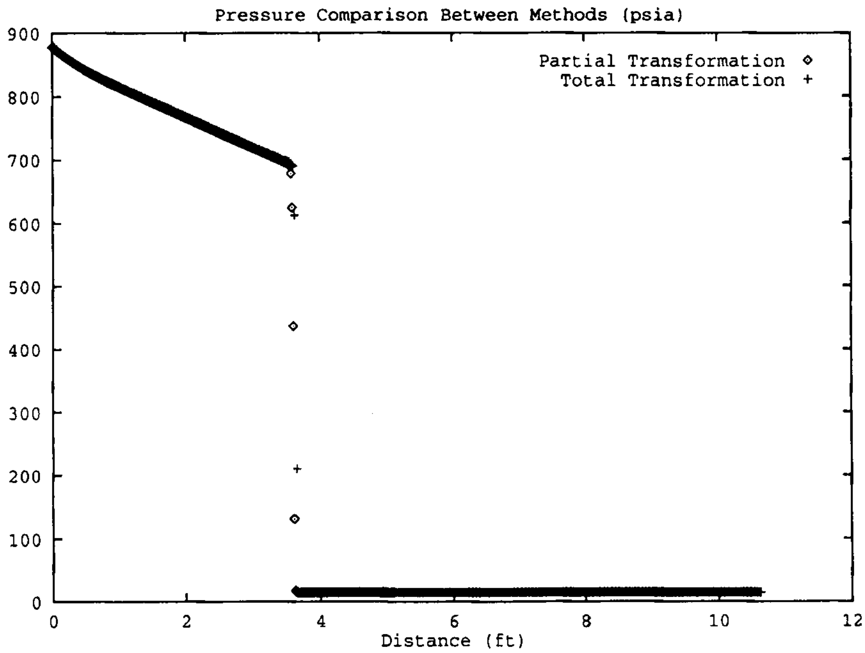

Figure 6. Direct comparison of pressure profiles for two schemes

6. THE 3D AXISYMMETRIC CASE

In this section the partial time transformation approach of Reference 2 is employed for a more realistic study of the internal ballistics problem previously considered. Results are presented for numerical simulations of the 3D, axisymmetric flow inside a 150 mm cannon, driven by an accelerating projectile. The previous piston velocity history is again used. However, the Godunov method of Reference 2 has been replaced with Harten's second-order TVD scheme, which is well-documented in Reference 4. A grid increment of 20 points per calibre, which is equivalent to that of the 1D calculation, is used.

Figures 7 and 8 show the bore-sight piston pressure build-up for the cases of blunt and non-blunt projectiles respectively. Here the non-planar piston consists of a 45°, 75 mm bevelled (staircase profile) appendage welded to the front of a 150 mm blunt projectile. Maximum bore-sight piston pressure comparison for the 1D and 3D problems gives respectively 878:850:841 psia, where the bevelled projectile slightly lowers the bore-sight pressure at the piston by flow push-off.

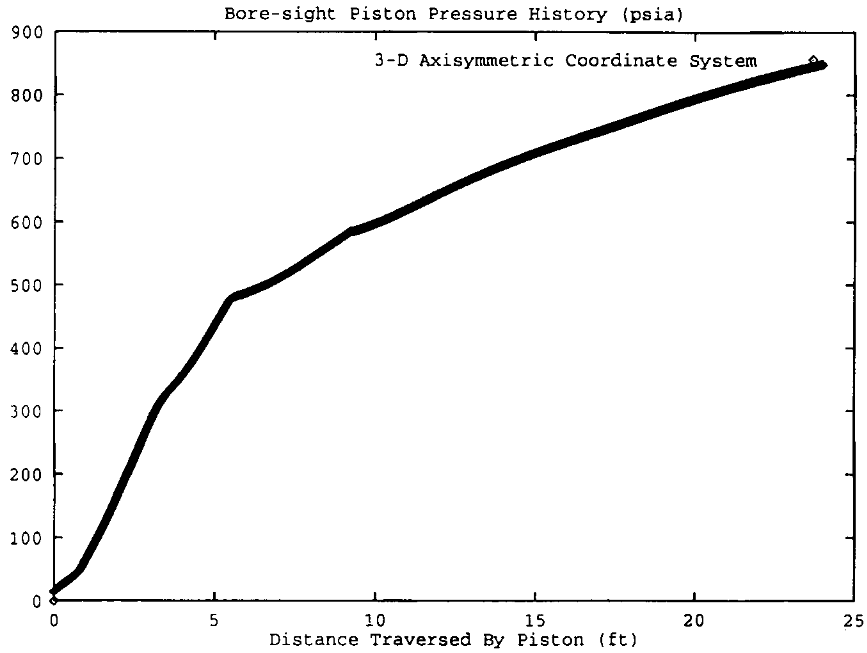

Figure 7. Axisymmetric bore-sight pressure build-up for a blunt projectile

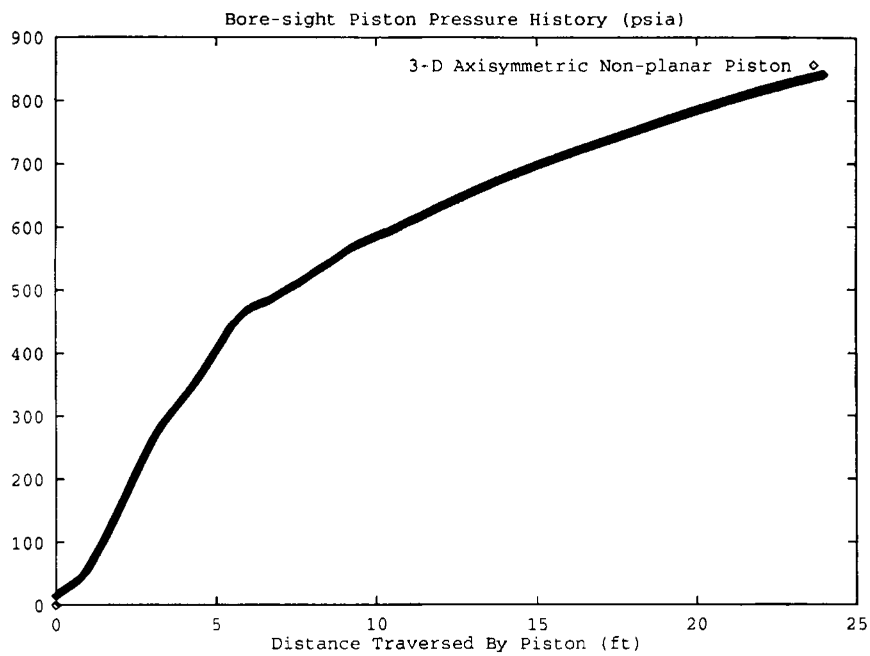

Figure 8. Axisymmetric bore-sight pressure build-up for a non-blunt projectile

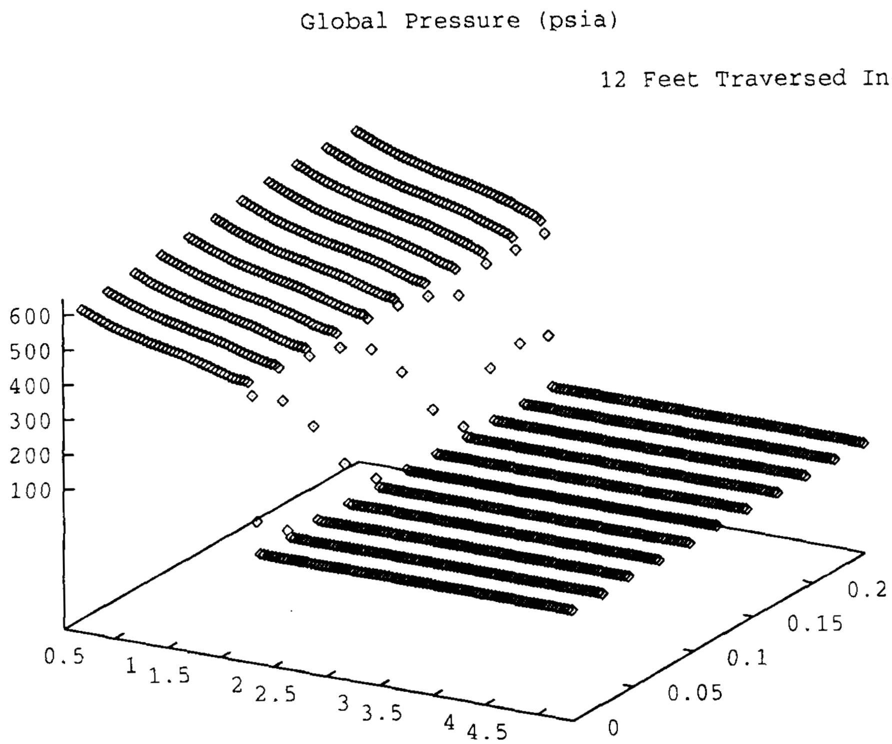

Figure 9. Global pressure

7. CONCLUSIONS

The transformation to non-Euclidean space coupled with operator splitting, Harten's second-order-accurate TVD scheme and the appropriate wall boundary condition appears to give good results for the supersonic sabot separation problem. At least, close agreement in results between the scheme introduced here and that of Reference 2 is realized. Most satisfying is the close agreement between results from the 1D and 3D axisymmetric flow models, which implies that for estimating the pressure build-up the extra cost of accounting for realistic in-bore flow and non-blunt projectiles is not really essential.

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